On the gauge and BRST invariance of the chiral QED with Faddeevian regularization

Anisur Rahaman
*Durgapur Govt. College, Durgapur - 713214, Burdwan, West Bengal, India**

Safia Yasmin
Indas Mahavidyalaya, Bankura - 722205, West Bengal, India

Sahazada Aziz
Burdwan University, Burdwan-713104, West Bengal, India

(Dated: June 26, 2009)

Chiral Schwinger model with the Faddeevian anomaly is considered. It is found that imposing a chiral constraint this model can be expressed in terms of chiral boson. The model when expressed in terms of chiral boson remains anomalous and the Gauss law of which gives anomalous Poisson brackets between itself. In spite of that a systematic BRST quantization is possible. The Wess-Zumino term corresponding to this theory appears automatically during the process of quantization. A gauge invariant reformulation of this model is also constructed. Unlike the former one gauge invariance is done here without any extension of phase space. This gauge invariant version maps onto the vector Schwinger model.

PACS numbers: 11.10.Ef, 11.30.Rd

I. INTRODUCTION

Symmetry plays a fundamental role in physics. Global symmetry imposes various constraint on the theory and the local symmetry leads to physical theory more realistic. Some times a given theory may be broken and that has a profound consequences. Gauge symmetry of a theory is of particular interest in this context. Absence of gauge symmetry invites anomaly in a theory. There have been considerable efforts in the understanding of anomaly in quantum field theory [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12]. The studies of chiral Schwinger model and anomalous Schwinger model are worth mentionable in this respect. It is the anomaly that removed the long suffering of chiral Schwinger model from non-unitarity. Credit went to Jackiw and Rajaraman - those who offered a consistent analysis of this model in a gauge non-invariant manner[1]. However a gauge invariant version is always favorable because of its increased symmetry. In the present work we are intended on restoration of gauge symmetry of chiral Schwinger mode [8, 9] with Faddeevian [5, 6] type of anomaly.

In terms of constraint [13], a gauge invariant theory is defined as a theory with first class constraint and a theory with second class constraint corresponds to anomalous theory. A powerful and elegant way to convert a theory with second class constraints to a theory with first class constraint was proposed by Batalin, Fradkin and Vilkovisky [14, 15, 16, 17, 18]. There have been attempts for this conversion in different approaches. The approaches basically falls into two independent classes. In one class extension of phase space through the introduction of auxiliary fields is required [14, 15, 16, 17, 18, 19, 20]. The other class however does not require this extension [21, 22].

Study of free chiral boson [4, 23, 24, 25, 26] as well as gauged chiral boson are very interesting in connection with the restoration gauge invariance because of its peculiar anomalous constraint structure. Gauge invariant reformulation of free chiral boson and gauged chiral boson are considered by several authors in diferent time [18, 19, 20, 21, 22, 27, 28]. It is known that two independent version of gauged chiral boson are available in the literature. The oldest one is the version proposed by Jackiw and Rajaraman [1]. We should mention here that Hagen initially gave the chiral generation of Schwinger model [29]. The model however failed to maintain the unitarity. Jackiw and Rajaraman saved the model introducing anomaly within it [1] and gave a consistent hamiltonian description of it. Mitra suggested an alternative gauge non-invariant version of gauged chiral boson [8, 9]. The anomaly of which corresponds to Faddeevian class of anomaly [5, 6] where gauss law constraint gave anomalous Poisson bracket among itself. The model attracted several attention because of this special type of anomaly structure. In spite of that the model rendered a consistent physically sensible result. It would be worthy to have a systematic development where gauge invariance gets restored and the Wess-Zumino term comes out automatically during the process. With this in view and also as a pedagogical

*Electronic address: anisur.rahman@saha.ac.in

illustration of the BVF formalism effort has been made to get a BRST invariant effective action of this model. The work will certainly demonstrate the power of BFV formalism once more. During the detailed study towards unitarity and renormalization of this model would this work may come in use.

The paper is organized as follows. Sec.II contains the bosonization of the fermionic version of chiral Schwinger model and imposition of a chiral constraint in it to express it in terms of chiral boson. In Sec. III, a BRST invariant reformulation of that model is carried through using Batalin , Fradkin and Vilkovisky formalism. Mitra-Rajaraman prescription is used in Sec. VI to obtain a gauge invariant reformulation of the same model. Sec. V contains a brief discussion over the work.

II. BOSONIZATION OF FERMIONIC MODEL AND IMPOSITION OF CHIRAL CONSTRAINTS

Chiral Schwinger model is described by the following generating functional

$$Z[A] = \int d\psi d\bar{\psi} e^{\int d^2x \mathcal{L}_f}, \tag{1}$$

with

$$\begin{aligned} \mathcal{L}_f &= \bar{\psi}\gamma^\mu[i\partial_\mu + e\sqrt{\pi}A_\mu(1-\gamma_5)]\psi \\ &= \bar{\psi}_R\gamma^\mu i\partial_\mu\psi_R + \bar{\psi}_L\gamma^\mu(i\partial_\mu + 2e\sqrt{\pi}A_\mu)\psi_L. \end{aligned} \tag{2}$$

The right handed fermion remains uncoupled in this type of chiral interaction. So integration over this right handed part leads to field independent counter part which can be absorbed within the normalization. Integration over left handed fermion leads to

$$Z[A] = exp[\frac{ie^2}{2}\int d^2x A_\mu[M_{\mu\nu} - (\partial^\mu + \partial^\mu)\frac{1}{\Box}(\partial^\nu + \partial^\nu)]A_\nu]. \tag{3}$$

$M_{\mu\nu} = ag_{\mu\nu}$, for Jackiw-Rajaraman regularization where the parameter a represents the regularization ambiguity and $M_{\mu\nu} = \begin{pmatrix} 1 & -1 \\ -1 & -3 \end{pmatrix}\delta(x-y)$, for an alternative version proposed in [8, 9].

Writing down the generating functional in terms of the auxiliary field $\phi(x)$ it turns out to the following

$$Z[A] = \int d\phi e^{i\int d^2x \mathcal{L}_B}, \tag{4}$$

with

$$\begin{aligned} \mathcal{L}_B &= \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + e(g^{\mu\nu} - \epsilon^{\mu\nu)})\partial_\nu\phi A_\nu + \frac{1}{2}e^2 A_\mu M^{\mu\nu}A_\nu \\ &= \frac{1}{2}(\dot{\phi}^2 - \phi'^2) + e(\dot{\phi} + \phi')(A_0 - A_1) + \frac{1}{2}e^2(A_0^2 - 2A_0A_1 - 3A_1^2). \end{aligned} \tag{5}$$

Here $\epsilon^{01} = -\epsilon_{01} = 1$ and the Minkowski metric $g^{\mu\nu} = diag\,(1,-1)$. From the standard definition, the momentum corresponding to the field ϕ is found out to be

$$\pi_\phi = \dot{\phi} + e(A_0 - A_1). \tag{6}$$

The following Legender transformation

$$H_B = \int d^2x[\pi_\phi\dot{\phi} - \mathcal{L}], \tag{7}$$

leads to the hamiltonian

$$\begin{aligned} \mathcal{H}_B &= \frac{1}{2}[\phi_\phi - e(A_0 - A_1)]^2 + \frac{1}{2}\phi'^2 - 2e\phi'(A_0 - A_1) \\ &- \frac{1}{2}e^2(A_0^2 - 2A_0A_1 - 3A_1^2). \end{aligned} \tag{8}$$

To suppress one chirality we impose the chiral constraint

$$\omega(x) = \pi_\phi(x) - \phi'(x) = 0, \tag{9}$$

at this stage. It is a second class constraint itself since

$$[\omega(x), \omega(y)] = -2\delta'(x-y). \tag{10}$$

After imposing the constraint $\omega(x) \approx 0$ into the generating functional we arrived at the following

$$\begin{aligned} Z_{CH} &= \int d\phi d\pi_\phi \delta(\pi_\phi - \phi') \sqrt{det[\omega, \omega]} e^{i\int d^2x(\pi_\phi \dot{\phi} - \mathcal{H}_B)} \\ &= \int d\phi e^{i\int d^2x \mathcal{L}_{CH}}, \end{aligned} \tag{11}$$

with

$$\mathcal{L}_{CH} = \dot{\phi}\phi' - \phi'^2 + 2e(A_0 - A_1)\phi' + 2e^2 A_1^2. \tag{12}$$

We obtained the gauged lagrangin for chiral boson from the bosonised Lagrangian with Faddeevian regularization just by imposing the chiral constraint in the phase space. Harada in [7], obtaind the same type of result for the usual chiral Schwinger model with one parameter class of regularization proposed by Jackiw and Rajaraman [1]. The lagrangian (12) can be thought of as the gauged version of chiral boson described by Floreanini and Jackiw [4]. The constraint analysis and the phase space structure corresponding to this model is available in [9]. In ref. [9], we found that the theory (12) describes a massive boson through the equation

$$[\Box + e^2]A_1 = 0 \tag{13}$$

with square of the mass $m^2 = e^2$. Equation (13) was interpreted there as the photon acquired mass and the fermion got confined.

III. BRST INVARIANT REFORMULATION USING BFV FORMALISM

BRST invariance essentially means to enlarge the Hilbert space of a gauge theory in order to restore the symmetry of a gauge fixed action in that enlarged space. It is very effective when one tries to study the renormalization property of a theory. One generally exploit the BRST symmetry instead of exploiting the original gauge symmetry. The discovery of this symmetry raised the ghost field to a prominent position. It mixes the ghost with the other fields of the theory and therefore all the fields including the ghosts can be regarded as a different components of a single geometrical object.

The combined formalism of Batalin, Fradkin and Vilkovisky [14, 15, 16, 17, 18] for quantization of a system is based on the idea that a system with second class constraint can be made effectively first class in the extended phase space which finally helps to find BRST invariant effective action. The field needed for this conversion ultimately turns out into the Wess-Zumino scalar with the proper choice of gauge condition, as pointed out by Fugiwara Igarashi and Kubo [16]. What follows next is a brief description of the general BFV formalism for obtaining a BRST invariant action.

Let us consider a canonical hamiltonian described by the canonical pairs $(p_i, q^i), i = 1, 2.......N$. The pairs are subjected to a set of constraints $\Omega_a \approx 0, a = 1, 2.......n$, and it is assumed that the constraints satisfy the following algebra.

$$[\Omega_a, \Omega_b] = i\Omega_c U^c_{ab}, \tag{14}$$

$$[H_c, \Omega_a] = i\Omega_b V^b_c, \tag{15}$$

then n no of additional condition $\Phi_a \approx 0$ with $det[\Phi_a, \Omega_b] \neq 0$ have to be imposed in order to single out the physical degrees of freedom. The constraints $\Omega_a \approx 0$ and $\Phi_a \approx 0$, together with hamiltonian equation of motion is obtained from the action

$$S = \int dt[p_i q^i - H_c(p_i, q^i) - \lambda^a \Omega_a + \pi_a \Phi^a], \tag{16}$$

where λ^a and π_a are Lagrangian multiplier field and these two satisfy the relation $[\lambda^a, \pi_b] = i\delta^a_b$.

Now introducing one pair of canonical ghost field $(C^a, \bar{P}_a)$ and one pair of canonical anti-ghost field $(P^a, \bar{C}_a)$ for each pair of constraints an equivalence can be made to the initial theory of the unconstrained phase space. So the quantum theory can be described by the partition function where the action in its numerator will be

$$S_{qf} = \int dt[p_i q^i + \pi^a \dot{\lambda}_a + \bar{P}^a \dot{C}_a + +\bar{C}^a \dot{P}_a - H_{BRST} + i[Q, \psi]. \tag{17}$$

H_{BRST} is the minimal hamiltonian as termed by Batalin and fradkin, is given by

$$H_{BRST} = H_c + \bar{P}_a V_b^a C^b. \tag{18}$$

The BRST charge Q and the fermionic gauge fixing function ψ are respectively given by

$$Q = C^a \omega_a - \frac{1}{2} C^b C_c U_{ab}^c - P^a \pi_a, \tag{19}$$

$$\psi = \bar{C}_c \chi^a + \bar{P}^a \lambda^a, \tag{20}$$

where χ_a's are expressed through the gauge fixing condition

$$\Phi_a = \dot{\lambda}_a + \chi_a \tag{21}$$

Let us now concentrate on the BRST invariant reformulation of the lagrangian (12). In order to do that we need to find out the constraint structure of the theory. To this end, we calculate the momenta corresponding to the fields A_0, A_1 and ϕ.

$$\pi_\phi = \phi', \tag{22}$$

$$\pi_1 = \dot{A}_1 - A_0', \tag{23}$$

$$\pi_0 = 0. \tag{24}$$

Note that the momenta $\pi_0 = 0$ and $\pi_\phi = \phi'$ are independent of velocity and these two are identified as the primary constraints of the theory. The constraints have to be preserve in time for the theory to be consistent and physically sensible. The preservation of the constraint $\pi_0 \approx 0$ leads to the Gauss law

$$G = \pi_1' + 2e\phi', \tag{25}$$

as the secondary constraint. Preservation of the constraint

$$\pi_\phi - \phi' = 0 \tag{26}$$

fixes the velocity v of the effective hamiltonian

$$H_{eff} = H_C + u\pi_0 + v\left(\pi\phi - \phi'\right), \tag{27}$$

where,

$$H_C = \pi_1^2 + \pi_1 A_0' + \phi^2 - 2e\left(A_0 - A_1\right)\phi' - 2e^2 A_1^2 \tag{28}$$

Velocity v is found out to be $v = \phi - e(A_0 - A_1)$. The preservation of the constraint (25) leads to the final constraint of the theory

$$A_1 + A_0 = 0. \tag{29}$$

The velocity u gets fixed by the preservation of the constraint (29) which comes out to be $u = -(\pi_1 + A_0')$. Therefore the theory under consideration contains four constraints in its phase space. Precisely the constraints are

$$\omega_1 = \pi_\phi - \phi' \approx 0, \tag{30}$$

$$\omega_2 = \pi_0 \approx 0, \tag{31}$$

$$\omega_3 = \pi_1' + 2e\phi' \approx 0, \tag{32}$$

$$\omega_4 = A_1 + A_0 \approx 0. \tag{33}$$

These four constraints form a second class set and the closures of the constrains with respect to the hamiltonian (27) are given by

$$\dot{\omega}_1 = \omega_1', \tag{34}$$

$$\dot{\omega}_2 = \omega_3 - \omega_2' + e\omega_1, \tag{35}$$

$$\dot{\omega} = \omega_4 - e\omega_1', \tag{36}$$

$$\dot{\omega}_4 = 2e^2\omega_2'. \tag{37}$$

To obtain a BRST invariant reformulation we need to convert the second class set of constraints into a first class set. With this in view, we introduce four auxiliary fields ψ , η, π_ψ, and π_η that satisfy the following canonical condition

$$[\eta(x), \pi_\eta(y)] = \delta(x-y), \tag{38}$$

$$[\psi(x), \pi_\psi(y)] = \delta(x-y). \tag{39}$$

The fields used here are known as Batalin Fradkin (BF) fields The constraints (30), (31), (32), and (33), with some suitable linear combination of the BF fields get converted into first class set as follows

$$\tilde{\omega}_1 = \pi_\phi - \phi' + \pi_\psi + \psi', \tag{40}$$

$$\tilde{\omega}_2 = \pi_0 - \pi_2, \tag{41}$$

$$\tilde{\omega}_3 = -2e\psi' + 2e\phi' + \pi_1' - \pi_\eta', \tag{42}$$

$$\tilde{\omega}_4 = -2e^2(A_0' - A_1') - 2e^2\eta'. \tag{43}$$

The consistency of the first class constraints with the first class hamiltonian will be maintained if these new first class set satisfy the same closures as their ancestor did with the hamiltonian (27). Precisely, the conditions are

$$\dot{\tilde{\omega}}_1, = \tilde{\omega}_1', \tag{44}$$

$$\dot{\tilde{\omega}}_2 = \tilde{\omega}_3 - \tilde{\omega}_2' + e\tilde{\omega}_1, \tag{45}$$

$$\dot{\tilde{\omega}}_3 = \tilde{\omega}_4 - e\tilde{\omega}_1', \tag{46}$$

$$\dot{\tilde{\omega}}_4 = 2e^2\tilde{\omega}_2'. \tag{47}$$

First class hamiltonian is obtained by the appropriate insertion of the BF fields within the hamiltonian (27) and it is given by $\tilde{H}$=H_P+H_{BF}. Here H_{BF} is a polynomial of ψ , η, π_ψ, and π_η that extend the phase space respecting the closures (44), (45), (46)and (47). We find that H_{BF} for this system will be

$$H_{BF} = -2e\eta\psi' + e(\pi_\psi + \psi') + \frac{1}{2}(\pi_\eta^2 + \pi_\psi^2 + {\psi'}^2). \tag{48}$$

We now introduce four pairs of ghost $(C_i, \bar{P}^i)$ and four pairs of ant-ighost $(P_i, \bar{C}^i)$ fields. Four pairs of multiplier fields (N^i, B_i) is also needed. The pairs satisfy the following canonical relations

$$\left[C_i, \bar{P}^j\right] = \left[P^i, \bar{C}_j\right] = \left[N^i, B_j\right] = i\delta^i_j\delta(x-y), \qquad i = 1, 2, 3, 4 \tag{49}$$

From the definition we can write BRST invariant hamiltonian

$$H_U = H_{BRST} - i[Q, \psi], \tag{50}$$

where H_U is the unitaring hamiltonian, Q is the BRST charge and ψ's are the gauge fixing funtions. Note that the BRST charge Q is a nilpotent operator and it satisfies the equation

$$Q^2 = [Q, Q] = 0. \tag{51}$$

The definition of Q in this formalism is

$$Q = \int (B_i P^i + C_i \tilde{\omega}^i) dx, \tag{52}$$

and the definition of gauge fixing function ψ is

$$\psi = \int (\bar{C}_i X^i + P_i N^i) dx. \tag{53}$$

The BRST invariant hamiltonian for the theory with which we are dealing with is

$$\begin{aligned} H_{BRST} &= H_P + H_{BF} + \int dx(-\bar{P}_1 C_1' + \bar{P}_3 C_2 + \bar{P}_2 C_2' + e\bar{P}_1 C_2 + \bar{P}_4 C_3 \\ &- e\bar{P}_1' C_3 + 2e^2 \bar{P}_2' C_4). \end{aligned} \tag{54}$$

It would be helpful to write down the generating functinal that ultimately leads to an effective action with the elimination of some fields by Gaussian integration. The generating funtional reads

$$Z = \int D(x) e^{iS}. \tag{55}$$

Here the expression of S is

$$\begin{aligned} S &= \int d^2x [\pi_\phi \dot{\phi} + \pi_1 \dot{A}_1 + \pi_0 \dot{A}_0 + \pi_\psi \dot{\psi} + \pi_\eta \dot{\eta} + \bar{P}_i \dot{C}^i + \bar{C}_i \dot{P}^i + B_i \dot{N}^i \\ &- H_{final}], \end{aligned} \tag{56}$$

where $H_{final} = H_U - i[Q, \psi]$, and $D\mu$ is the Liouville measure in the extended phase space. We are now in a position to fix up the gauge condition.

$$\chi_1 = \pi_\phi - \phi',, \tag{57}$$

$$\chi_2 = -\dot{N}^2 + A_0,, \tag{58}$$

$$\chi_3 = \frac{B_3}{2} - A_1',, \tag{59}$$

$$\chi_4 = \pi_\eta - \dot{N}^4.. \tag{60}$$

When we substitute the simplified form of H_{final} obtained after plugging fixing conditions (57), (58), (59) and (60) in the action (56), we get the explicit expression of S.

$$\begin{aligned} S &= \int d^2x [\pi_\phi \dot{\phi} + \pi_\psi \dot{\psi} + \pi_\eta \dot{\eta} + \pi_1 \dot{A}_1 + \pi_0 \dot{A}_0 + \bar{P}_i \dot{C}^i + \bar{C}_i \dot{P}^i + B_i \dot{N}^i \\ &- (\frac{\pi^2}{2} + \pi A_0' + e\phi'(A_0 - A_1) + 2e^2 A_1^2 + \pi_0(-\pi_1 - A_0') + \phi' \pi_\phi \\ &- e\pi_\phi(A_0 - A_1) - e\eta\psi' + e\pi_\psi \eta + \frac{1}{2}(\pi_\eta^2 + \pi_\psi^2 + \psi'^2) + B_i X^i + \tilde{\omega}_i N^i \\ &- \bar{P}_i P^i - \bar{P}_1 C_1' + \bar{P}_3 C_2 + \bar{P}_2 C_2' + e\bar{P}_1 C_2 + \bar{P}_4 C_3 - e\bar{P}_1 C_3 + 2e^2 \bar{P}_2' C_4 \\ &- P_1 \bar{P}^1 - P_2 \bar{P}^2 - P_3 \bar{P}^3 - P_4 \bar{P}^4 - C_3 \bar{C}'' - \bar{C}_2 \dot{P}^2 - \bar{C}_4 \dot{P}^4 + 2e^2 C^4 \bar{C}_4 \\ &- 2C' \bar{C}_1 + e^2 C^2 \bar{C}_2)]. \end{aligned} \tag{61}$$

Here i runs from 1 to 4. Our next tusk is to simplify equation (55) through the elimination of some fields and that will lead us to our desired result. A careful look reveals that here exists a simplification

$$\int d^2x(B_iN^i + \bar{C}_i\dot{P}^i) = -i[Q, \int d^2x(\bar{C}_i\dot{N}^i)] \tag{62}$$

with be legendre transformation B^i goes to $B^i + \dot{N}^i$. However the simplification corresponding to $i = 1$ suffices in this situation. More simplification follows from the elimination of the fields $\pi_0,\ \pi_1,\ \pi_\eta,\ B_1, B_2, B_4, A_0, N^1, N^2, N^4, P_1, \bar{P}^1, P_2,\ \bar{P}^2, P_4, \bar{P}^4, P_1, \bar{P}^1,\ C_1, \bar{C}^1, C_2$ and , $\bar{C}^2$ by Gaussian integration. Ultimately we reach to a very simplified form of the generating functional (55) that contains the following effective action in its numerator.

$$\begin{aligned} S_{eff} &= \int d^2x(\dot{\phi}\phi' - \phi'^2 + 2e^2A_1^2 - \psi'^2 - \dot{\psi}\psi' + \frac{\dot{A}_1 - A_0')^2}{2} \\ &+ 2e\phi'(A_0 - A_1) + 2e\psi'(A_1 - A_0) + \partial_\mu BA^\mu + \frac{1}{2}\alpha B^2 \\ &+ \partial_\mu C^3\partial_\mu\dot{C}^3 \end{aligned} \tag{63}$$

We have used few redefinition of fields, e.g., $N_3 = A_0$ and $P^3{=}\dot{C}_3$ to reach to the result (63). Since after elimination there is no other B's and C's except B_3 and C_3 we are free to read them as B and C. It is now time to check the invariance of the action (63). A little algebra shows that the action is invariant under the transformation

$$\begin{aligned} \delta A_1 &= -\lambda C', \delta A_0 = \delta N_3 = \lambda C_1 \\ \delta\psi &= \lambda C_1, \delta C = \lambda B, \delta C = 0 \end{aligned} \tag{64}$$

It is to be mentioned that the fields satisfy the following Weller-lagrange equation

$$\partial_-\phi - \partial_+\psi - 2eA_1 = 0 \tag{65}$$

We can identify easily the Wess-Zumino term for this theory which is

$$\mathcal{L}_{wz} = -\dot{\psi}\psi' - \psi'^2 - 2e\psi(A_0 - A_1) \tag{66}$$

It is interesting to see this automatic appearance of this Wess-Zumino term during the process of obtaining the BRST invariant action. One point I should mention here that the of choice of gauge condition is very crucial. One may miss to get Wess-Zumino term other wise.

IV. GAUGE INVARIANT REFORMULATION WITH OUT EXTENDING THE PHASE SPACE

The formalism of making a theory gauge invariant by the reduction of of the number of second class constraint was first developed by Mitra and Rajaraman [21, 22]. The formalism strictly depends on the constraint structure of the theory. Depending on the constraint structure of the theory different gauge invariant version is possible for a particular theory. No extension of phase space is needed in this formalism. So the physical contents of all the gauge invariant actions remains the same. The thumb rule of this formalism is to reduce half of the constraint from a second class constraint set retaining the first class set only. We have already seen that the phase space of the chiral schwinger model with Faddevian anomaly contains the following four constraints.

$$\omega_1 = \pi_\phi - \phi' \approx 0, \tag{67}$$

$$\omega_2 = \pi_0 \approx 0, \tag{68}$$

$$\omega_3 = \pi_1' + 2e\phi' \approx 0, \tag{69}$$

$$\omega_4 = A_1 + A_0 \approx 0. \tag{70}$$

Note that the combination $\omega_2 \approx 0$ and $\omega_3 \approx 0$ form a first class set. If we retain these two constraints eliminating other two following Mitra-Rajaman prescription it requires a modification of the hamiltonian of the second class system in the following manner

$$\begin{aligned}\mathcal{H} &= \frac{1}{2}\pi_1^2 + \pi_1 A_0' - e(A_0 - A_1)\phi' + 2e^2 A_1^2 + \pi_\phi \phi' - e\pi_\phi(A_0 - A_1) \\ &+ e(\pi_\phi - \phi')(A_0 - A_1) + \frac{(\pi_\phi - \phi')^2}{2}. \end{aligned} \tag{71}$$

The modification certainly keeps the physical contents of the theory intact. It easy to check that the modified hamiltonian (71) contains only the two first class constraints $\omega_2 \approx 0$ and $\omega_3 \approx 0$. Now the equation of motion with respect to the hamiltonian (71) are found out as follows

$$\dot{\phi} = 2eA_1 + \pi_\phi, \tag{72}$$

$$\dot{A_0} = -u, \tag{73}$$

$$\dot{A_1} = \pi_1. \tag{74}$$

A straightforward calculation leads to the lagrangian corresponding to the first class theory with which we are interested in.

$$\begin{aligned}\mathcal{L}_1 &= \pi_\phi\dot{\phi} + \pi_1\dot{A_1} + \pi_0\dot{A_0} - [\frac{\pi_1^2}{2} + \pi_1 A_0' + 2eA_1\pi_\phi + \pi_\phi\phi' - 2eA_0\phi' \\ &+ \frac{(\pi_\phi - \phi')^2}{2} + u\pi_0 + 2e^2A_1^2]. \end{aligned} \tag{75}$$

After a little algebra the lagrangian acquires a very simplified firm

$$\mathcal{L}_2 = \frac{1}{2}(\dot{\phi}^2 - \phi'^2) - 2e(A_1\dot{\phi} - A_0\phi') + \frac{1}{2}(\dot{A_1} - A_0')^2. \tag{76}$$

The lagrangian (76), is consistent with the hamiltonian (71), and the equations of motion (72), (73) and (74). It has only two first class constraints (68) and (69) in its phase space which help us to construct gauge transformation generator. The generator is given by

$$\mathcal{G} = \int dx(\lambda_1\omega_1 + \lambda_2\omega_2) \tag{77}$$

λ_1 and λ_2 are two arbitrary parameter. The transformations evolved out of the generator (77) for the fields ϕ, A_1 and A_0 respectly are

$$\delta\phi = 0, \delta A_1 = -\lambda_1', \delta A_0 = -\lambda_2. \tag{78}$$

A little algebra shows that under the transformation (78), the lagrangian (76) remains invariant provided the parameter satisfy the relation

$$\lambda_2 = \dot{\lambda_1} \tag{79}$$

A note worthy thing is that this transformation is equivalent to the transformation $A_\mu \leftarrow A_\mu + \frac{1}{e}\partial_\mu\lambda$. There is some thing interesting that we must mention here. The first class lagrangian that comes out from our investigation is the bosonised lagrangian of the well known vector Schwinger model [30, 31]. It is not of so surprising because the spectrum of the model under consideration is identical to the vector Schwinger model. To be precise, both the models contain the massive boson with the square of the mass $m^2 = e^2$. We have mentioned earlier that the gauge invariant reformulation follows from this prescription depends crucially on the constraint structure of the model and that allow some other possibilities to get first class set of constraints from the set of constraints (67), (67), (67) and (70). However that possibilities fail to give consistent first class theories.

V. DISCUSSION

Gauge invariant reformulation of chiral Schwinger with Faddeevian anomaly has been carried through in two different directions. In the first case BFV prescription [14, 15, 16, 17, 18] is followed which needs an extension of phase space. The process certainly keeps the physical contents of the theory intact. The fields needed for the extension keep themselves allocated in the unphysical sector of the theory. In this prescription we not only get a BRST invariant effective action but also appropriate Wess-Zumino term appears automatically during the process. In the second approach Mitra-Rajaraman [21, 22] prescription is followed to obtain a gauge invariant action. In this situation we have to be restricted on the gauge invariance only because the formalism developed till now is not adequate to obtain BRST invariant action. As we have already mentioned that in spite of more than one possibilities only a particular possibility leads to a gauge invariant action. Surprisingly, the other possibilities fail to do so. Only that possibility has explored to obtain gauge invariant reformulation which renders a very interesting result. The gauge invariant model that comes out is the lagrangian of well known vector Schwinger model [30, 31] and gauge invariance of which is obvious. Physical contents of this gauge invariant version is identical to the vector Schwinger model [30, 31]. The counting of degrees of freedom also gives a consistent result. It would be interesting to investigate how a particular version of the chiral Schwinger model maps onto the vector Schwinger model in its gauge invariant version.

[1] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. **54** 1219 (1985)
[2] H. O. Girotti, H. J. Rothe and K. D. Rothe, Phys. Rev. **D33** 514 (1986)
[3] H. O. Girotti, H. J. Rothe and K. D. Rothe, Phys. Rev. **D34** 592 (1986)
[4] R. Floreanini and R. Jackiw, Phys. Rev. Lett. **59** 1873 (1987)
[5] L. D. Faddeev, Phys. Lett. **B154** 81 (1984)
[6] L. D. Faddeev and S. L. Satashvili, Phys. Lett. **B167** 225 (1986)
[7] K. Harada, Phys. Rev. Lett. **64** 139 (1990)
[8] P. Mitra, Phys. Lett. **B284** 23 (1992)
[9] S. Ghosh and P. Mitra, Phys. Rev. **D44** 1332 (1990)
[10] P. Mitra and A. Rahama, Ann. Phys. (N. Y.) **249** 34 (1996)
[11] A. Rahaman, Int. J. Mod. Phys. **A12** 5625 (1997)
[12] A. Rahaman, Int. J. Mod. Phys. **A19** 3013 (2004)
[13] P. A. M. Dirac, Lectures on quantum Mechanics (Yeshiva Univ. Press, New York, 1964)
[14] E. S. Fradkin and G.A. Vilkovisky, Phys. Lett. **B55** 224 (1975)
[15] I. A. Batalin and E. S. Fradkin, Nucl. Phys. **B279** 514 (1987)
[16] T. Fujiwara, I. Igarashi and J. Kubo, Nucl. Phys. **B314** 695 (1990)
[17] I. A. Batalin and V. Tyutin, Int. J. Mod. Phys. **A6** 3255 (1991)
[18] Y. W. Kim S. K. Kim, W. T. Kim, Y. J. Park, K. Y. Kim and Y. kim, Phys. Rev. **D46** 4574 (1992)
[19] S. J. Yoon, Y. W. Kim and Y. J. Park, J. Phys. **G25** 1783 (1989)
[20] M. I. Park, Y. J. Park, and S. J. Yoon, J. Phys. **G24** 2179 (1988)
[21] P. Mitra and R. Rajaraman, Ann. Phys. (N. Y.) **203** 137 (1990)
[22] P. Mitra and R. Rajaraman, Ann. Phys. (N. Y.) **203** 157 (1990)
[23] W. Siegel, Nucl. Phys. **B238** 307 (1984)
[24] S. Bellucci, M. F. L. Golterman and D. N. Petcher, Nucl. Phys. **B326** 307 (1989)
[25] C. Imbimbo and J. Strominger, Phys. Lett. **B193** 445 (1987)
[26] J. M. F. Labastida and M. permici, Nucl. Phys. **297** 557 (1988)
[27] S. Ghosh, Phys. Rev. **D49** 2990 (1994)
[28] Y. G. Miao, J. G. Zhou and Y. Y. Liu, J. Phys. **G19** 1797 (1993)
[29] C. R. Hagen, Ann. Phys. (N. Y.) **81** 67 (1973)
[30] J. Schwinger, Phys. Rev. **128** 2425 (1962)
[31] J. H. Lowenstein and J. A. Swieca, Ann. Phys. (N. Y.)**68** 172 (1971)